# Optimizing BNF Grammars through Source Transformations 

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#### Abstract

In this paper we explore the efficiency of various ways of expressing the form and meaning of natural language utterances as context-free grammars. We concentrate on the top-down parsing strategy employed in SpeechWorks 6.5, a strategy common to many systems. As with other search-based parsers, the key to efficiency is to limit the uncertainty of the parser at any given stage by reducing non-determinism in the grammar. Here we study the effects of different expressions of the same grammar in terms of efficiency. We also describe a methodology for transforming a source grammar into a more efficient expression of the same forms and meanings.


## INTRODUCTION

Sophisticated spoken dialogue applications rely on the ability to interpret a wide range of user utterances. The usual result is a complex natural language grammar expressing both what someone can say to the system and what the system will understand them to mean. Complex grammars can lead to bottlenecks in processing if they are not expressed in an efficient way. This paper concentrates on the notion of parsing as computation in which the programs are grammars. As with other forms of programming, grammars expressing the same thing can be encoded in more or less efficient forms. As with most other problems in spoken language recognition and understanding, the fundamental principle is the reduction of nondeterminism. In this paper, we will show how this can be accomplished in a principled way.

We begin with a survey of the SpeechWorks architecture for recognition and interpretation. Next, we provide the standard algorithm for top-down search-based parsing. We briefly discuss the role of semantic interpretation, and then move on to a description of how grammars can be optimized for top-down search-based parsing. We provide empirical evidence in terms of comparative run times for simple, equivalent grammars. We conclude with a discussion of general techniques for optimizing grammars for top-down search-based parsers.

## RECOGNITION AND PARSING PASSES

The SpeechWorks 6.5 architecture [1] for extracting meaning from user utterances involves three passes, including one acoustic and statistical language model pass and two context-free grammar-based passes.

First Pass: Decoding Operates forward to construct a word graph. Scores are determined by successively refined acoustic models and a bigram language model.

Second Pass: Parsing Operates backwards to extract parses for the $n$-best word string hypotheses that are accepted by a specified context-free grammar.

Third Pass: Interpretation Evaluates semantics for parses generated in the second pass. Grammars are allowed to re-score hypotheses on the basis of semantics. ${ }^{1}$ Word strings with equivalent semantics are conflated before confidence scores are computed.

The final result to be processed by the dialog engine is a ranked list of semantic hypotheses in the form of key-value pairs. Subsequent processing can re-score these hypotheses further using context-specific information, such as user profiles, time of day, availability of requested information, etc.

## CONTEXT-FREE GRAMMARS

The context-free grammar paradigm has been thoroughly studied in the formal languages and automata theory literature (for example, see [2] for a range of definitions and theorems). A context-free grammar is essentially a finite collection of phrase structure rules. More formally, we assume a set Word of words (often called terminals), a set NonTerminal of nonterminals, and a finite set Rule of grammar rules, where each grammar rule is of the form $(\mathrm{C}=>\mathrm{X} 1 . . \mathrm{Xn})$ where C is a nonterminal and each of the $\mathrm{X} i$ is either a non-terminal or a word.

Each grammar defines a set of parse trees. The set of trees, Tree, is defined to be the least set such that Word $\subseteq$ Tree and such that $[\mathrm{C} \mathrm{T1} \ldots \mathrm{~T} n] \in$ Tree if $\mathrm{C} \in$ NonTerminal, $n \geq 0$, and $\mathrm{T} 1, \ldots \mathrm{~T} n \in$ Tree. For a tree [C T1 ...Tn], the nonterminal C is said to be the root. The yield of a tree is defined by $\operatorname{yield}(w)=w$ if $w$ is a word, and $\operatorname{yield}([\mathrm{C} \mathrm{T1} \ldots \mathrm{~T} n]=$ yield $(\mathrm{T} 1)$. . yield $(\mathrm{T} n)$; this is essentially the sequence of words appearing in the tree read left to right. A parse tree is a tree every node of which was derived by a rule. More formally, a tree $[\mathrm{C} \mathrm{T1} \ldots \mathrm{~T} n]$ is a parse tree if $(\mathrm{C} \Rightarrow \operatorname{root}(\mathrm{T} 1) . . \operatorname{root}(\mathrm{T} n))$ is a grammar rule and $\mathrm{T} 1, \ldots, \mathrm{~T} n$ are themselves parse trees or

[^0]single words. If we take the yields of all of the parse trees for a grammar rooted at a given non-terminal C (often called the start symbol or root category), we have what is known as a contextfree language (a particular kind of formal language).

## PARSING

Parsing is essentially the task of determining for a given string of words what the valid parse trees are with that string as yield. Obviously, only strings in the language will have parse trees, and thus parsers can be used to determine whether a string is in the language of a grammar.

Top-down search-based parsing (also known as recursive descent parsing) is a widely used method for parsing both computer languages [3] and natural languages [4]. The primary difference between computer languages and natural languages is that natural language grammars typically admit ambiguity. A typical kind of ambiguity involves attachment of modifiers, as in the contrast between the attachment of the prepositional phrases in the following pair of trees that yields the string she saw the boy with the telescope: [ $\mathrm{S}[\mathrm{NP}$ she] [VP saw [ NP the $[\mathrm{N}[\mathrm{N}$ boy] [PP with the telescope $]]]]]$ versus [ $\mathrm{S}[\mathrm{NP}$ she] $[\mathrm{VP}$ [VP saw [NP the [N boy]]] [PP with the telescope]]] (meaning in the first case that the boy had the telescope and in the second case, that she did).

The simplest way to define any search-based algorithm is to provide the search space (see [4], for example). A search space is characterized by a mapping from an input to a start state, a collection of final states, and a transition function that takes you from one state to another. For top-down search-based parsing, the states represent how much of the input string is left and which non-terminals need to be found. Suppose we have an input string $W s$ that we are trying to parse into a tree rooted at non-terminal S (the start symbol) with respect to a given grammar. Each state in the search space will be of the form Ws / Cs where Ws is a sequence of words and Cs is a sequence of non-terminals. ${ }^{2}$ The search space is. ${ }^{3}$

Start State: Ws / [S]
Final State: []/[]
Expand: Ws / [C0|Cs] => Ws / [C1,..,Cn|Cs]
[if $\mathrm{C} 0 \Rightarrow \mathrm{C} 1, \ldots, \mathrm{Cn}$ a rule of the grammar]
Match: $[\mathrm{W} \mid \mathrm{Ws}] /[\mathrm{W} \mid \mathrm{Cs}]=>\mathrm{Ws} / \mathrm{Cs}$
${ }^{2}$ We use the Prolog notation for lists, where [W|Ws] is a list whose first element is W and where Ws is a list consisting of the remaining elements in the list. For instance, the list a,b,c would be represented as $[\mathrm{a} \mid[\mathrm{b} \mid[\mathrm{c} \mid[]]]]$ where [ ] is the empty list containing no elements. We will also write [a,b,c] and $[\mathrm{a}, \mathrm{b} \mid[\mathrm{c} \mid[]]$ for the same list. ${ }^{3}$ SpeechWorks' parser actually operates right-to-left rather than left-to-right, but we adopt the usual directional convention here; the empirical data for left and right recursion are also reversed for consistency with this paper.

The list of categories represents things we still need to find, whereas the list of words represents the words remaining to consume. Thus we start in the state Ws / [S] with all of the input words remaining and the start-state as the only nonterminal being sought. The final state represents the situation in which we have found all of the categories we were looking for and have consumed all of the input in so doing. The two transitions in this space correspond to the top-down expansion of a grammar rule and the matching of input to the grammar. We can expand a category that we are looking for by replacing it with its daughters in a rule. Similarly, if we are looking for a word and we have that word at the beginning of the remaining words, we can consume it.

Example: Suppose we have a very simple grammar for English sentences containing rules $\mathrm{S} \Rightarrow \mathrm{NP}$ VP; NP $=>$ Det N ; Det $=>$ the; $\mathrm{N}=>k i d ; \mathrm{VP}=>\mathrm{V} ; \mathrm{V} \Rightarrow$ ran. Then if we are parsing the string the kid ran, we have the following sequence of states:

| [the, kid, ran] / [S] | start state |
| :---: | :---: |
| [the, kid, ran] / [NP, VP] | expand $\mathrm{S} \Rightarrow \mathrm{NP}$ VP |
| [the, kid, ran] / [Det, N, VP] | expand NP $=>$ Det N |
| [the, kid, ran] / [the, N, VP] | expand Det => the |
| [kid, ran]/ [N, VP] | match the |
| [kid, ran]/ [kid, VP] | expand $\mathrm{N}=>$ kid |
| [ran]/ [VP] | match kid |
| [ran]/[V] | expand VP $=>\mathrm{V}$ |
| [ran]/ [ran] | expand $V=>$ ran |
| []/[] | match ran |

As in speech recognition, we are typically interested in all the possible parses for a given input string. ${ }^{4}$ As such, we will need to do an exhaustive search of the search space (usually known as all-paths parsing). Note that a path through this search space uniquely determines a parse tree. This tree can either be constructed from the steps taken during search, or it can be built online each time a rule is expanded. Either way, it is the final parse tree from which the semantics will be computed. Typically, some degree of lexical lookahead is allowed rather than just blindly expanding rules top-down hoping to hit upon the right lexical item; this is achieved by only expanding rules whose right-hand sides begin with words if the appropriate word is there in the input.

Typically, top-down search-based parses exclude certain forms of problematic grammars from consideration. The primary candidates for exclusion are the left-recursive grammars,
${ }^{4}$ We will not be discussing probabilistic parsing and the pruning that usually goes along with it in this paper (see [4] for an introduction); but we should point out that our optimizations also apply to probabilistic parsers with beam search.
because they introduce infinite loops into the search space. ${ }^{5}$ For instance, the left-recursive rule $\mathrm{N}=>\mathrm{N}$ PP allows a noun to be followed by a prepositional phrase; this means that state $\mathrm{Ws} /[\mathrm{N} \mid \mathrm{Cs}]$ expands to $\mathrm{Ws} /[\mathrm{N}, \mathrm{PP} \mid \mathrm{Cs}]$, which in turn produces $\mathrm{Ws} /[\mathrm{N}, \mathrm{PP}, \mathrm{PP} \mid \mathrm{Cs}]$ and so on ad infinitum. A grammar is said to be left recursive if there is a sequence of rules ${ }^{6} \mathrm{~A} 0 \Rightarrow$ $\mathrm{A} 1 . \therefore \mathrm{A} 1 \Rightarrow \mathrm{~A} 2 . ., \ldots ; \mathrm{A} n-1=>\mathrm{A} n . .$. Such that $\mathrm{A} 0=\mathrm{A} n$. So $\mathrm{N}=>\mathrm{N} P P$ is left recursive with $n=1$, and the pair $\mathrm{A}=>\mathrm{B} \mathrm{C}, \mathrm{B}$ $\Rightarrow \mathrm{AD}$ is left recursive with $n=2$. Left recursive grammars form the limit case of top-down non-determinism, allowing an infinite number of states to be reachable from another state without consuming any input. In general, we define the degree of ambiguity for a given state to be the number of states that are reachable from it without consuming any input. Left recursive grammars have an infinite degree of ambiguity. If a grammar is not left recursive, every state has a finitely bounded degree of ambiguity. The complexity of search-based parsing is directly proportional to the number of states that are reachable from the initial state. If every state has a finite degree of ambiguity, at least that number will be finite. Our goal is to reduce the number of states as much as possible without damaging the parse tree topology beyond our ability to reconstruct the semantics from it.

## A SIMPLE CASE

Using the parser from SpeechWorks 6.5, we tested the efficiency of three forms of representing the same grammar. For the sake of simplicity, we focused on a very common and easily understood grammar - that for allowing a sequence of between 1 and N instances of a given non-terminal A . We considered three ways such a grammar might be written. ${ }^{7}$

## Disjunctive Form

S $\Rightarrow \mathrm{A} ; \mathrm{S}$ => A A; S $\Rightarrow$ A A A; S => A A A A; $\ldots$

## Left "Recursive" Form

$\mathrm{S} \Rightarrow \mathrm{A} n ; \mathrm{A} n \Rightarrow \mathrm{~A} ; \mathrm{A} n=\mathrm{A} n-1 \mathrm{~A} ; \mathrm{A} n-1 \Rightarrow \mathrm{~A} ; \mathrm{A} n-1 \Rightarrow$ $\mathrm{A} n-2 \mathrm{~A} ; . . ; \mathrm{A} 1 \Rightarrow \mathrm{~A}$

## Right "Recursive" Form

$\mathrm{S} \Rightarrow \mathrm{A} n ; \mathrm{A} n=>\mathrm{A} ; \mathrm{A} n \Rightarrow \mathrm{~A} \mathrm{~A} n-1 ; \mathrm{A} n-1 \Rightarrow \mathrm{~A} ; \mathrm{A} n-1 \Rightarrow$ A $\mathrm{A} n-2 ; . \therefore \mathrm{A} 1=>\mathrm{A}$

[^1]Theoretical analysis of the degree of ambiguity of these grammars is reflected in their empirical run times. For both the disjunctive and the left "recursive" grammar (it' s not truly recursive - just a finite approximation of a recursive grammar), parsing a string of As of length $m$ against a grammar that accepts from 1 to $n$ instances of non-terminal A results in a number of states on the order of $\boldsymbol{O}(n \mathrm{~m})$ to be explored (see [2] or [5] for a definition of the $\boldsymbol{O}$ notation). For instance, in the disjunctive case, there are $n$ states expanded before the first input is consumed, $n$ - 1 states expanded at step $2, n-2$ states expanded at step 3 , and so on up to $n-m$ states expanded after consuming $m$ inputs, and $(n+(n-1)+\ldots+(n-m))$ is $\boldsymbol{O}(n m)$. The left "recursive" case is slightly worse. On the other hand, the number of states explored in the right "recursive" grammar is linear in the input, or $\boldsymbol{O}(m)$. This is because there are at most two states explored per input token in the right "recursive" grammar, for a total of $2+2+\ldots+2$ ( $m$ times), which is clearly $\boldsymbol{O}(m) .{ }^{8}$ We do not consider the doubly recursive grammar of the form A $=>$ A A, which allows any parse tree to be generated; its complexity is exponential (its given by the Catalan numbers [5]). This case does arise in natural language through noun compounding, where $\mathrm{N} \Rightarrow \mathrm{N} \mathrm{N}$, as in $[\mathrm{N}[\mathrm{N}[\mathrm{N}[\mathrm{N}$ towel $]$ [ N rack]] [ N designer $]$ [ N [ N training] [ N courses $]]$ ].

Our empirical results support the theoretical analysis. Using the SpeechWorks 6.5 parser, we explored grammars accepting between 1 and $n$ words from a given non-terminal category A. In Figure 1, we show the results for the three grammars above. ${ }^{9}$ We consider the case where the non-terminal A expands to a set of 256 words, testing against 5000 randomly generated test with uniformly distributed lengths. Note that the quadratic codings take roughly 50 times as long to process in longer cases.

## A REALISTIC CASE: COUNTING

Although the above case was designed for illustrative simplicity, we have encountered similar cases in real applications. One example from a real grammar involves agreement between a keyword and the number of instances. A precise grammar for this case is: $\mathrm{S}=>\mathrm{A}$ one; $\mathrm{S}=>$ A A two; $\mathrm{S}=>$ A A A three; .... The actual example involved a non-trivial expansion of the nonterminal A , and a number of subcases, but the above grammar illustrates the point. As we saw in Figure 1, simply using the grammar as given above is not particularly efficient. The obstacle in the way of directly implementing the right recursive approach above is that we need to count the number of occurrences and pick up an agreeing word at the end. In order to do this, we can use a grammar of the following form, in which we

[^2]represent the count as we proceed down the tree and make sure we pick up an agreeing element later on. This is achieved with the grammar: $\mathrm{S}=>\mathrm{A} \mathrm{S1;} \mathrm{S1}=>$ one; S 1 => A S2; S2 $=>$ two; S2 $=>$ A S3; ...The trick is to simply use the non-terminals as counters. One problem is that the resulting tree structure is not what one might expect, but rather looks like [S [A ..] [S1 [A ..] [S2 two $]$ ]]. In particular, this can cause problems for the propagation of semantics. Fortunately, this problem is easily solved if the grammar formalism allows semantic rules to be attached to the grammar rules, as in standard compilers (see [3]). For instance, if the word "two" has some distinguished semantic value, this can simply be propagated by the rules for $\mathrm{S}, \mathrm{S} 1$, and S 2 . In general, this problem can be handled by a technique known in the programming literature as continuation passing (see, for example, [6] or [7]).

## THE GENERAL CASE

In general, our goal can be seen as simply reducing nondeterminism. We can apply two kinds of transforms to our grammars to convert them to equivalent forms: folding and unfolding, techniques that are widely employed in optimizing compilers (see [3]). An unfolding is a particular case of a general technique known as partial evaluation in which one rule is expanded inside of another. For instance, it would take A $=>$ B C D and $\mathrm{C}=>\mathrm{F}$ that E and produce $\mathrm{A} \Rightarrow \mathrm{B} F$ that E D . A frequent general case of folding is applied when there are multiple rules that share the same prefix. In particular, we look for rules with common prefixes and collapse them. For instance, if we have two rules A => B C D and A $\Rightarrow>B C E$, then every time we are looking for an A we expand both rules. We can fold the repeated sequence $\mathrm{B} C$ together with a rule $\mathrm{B} \_\mathrm{C} \Rightarrow>\mathrm{B}$ C and replace the two rules above with $A=>B \_C D$ and $A=>$ B_C E. Now only one rule is expanded. If there is a lot of this kind of duplication, as in our disjunctive grammar above, the savings can be significant.

Folding and unfolding can be used to normalize grammars. For instance, every grammar can be converted to an equivalent grammar with no left recursion, the so-called Greibach normal form, in which every rule begins with a word; that is, is of the form $\mathrm{A}=>\mathrm{b} \mathrm{C1}$.. Cn, for $n \geq 0$.

Unfortunately, it is impossible to fully determinize all con-text-free grammars (see [2]); some are intrinsically ambiguous. Even the simple problem of context-free grammar equivalence is undecidable [2]. But we can automatically detect a lot of sharing, and unfold an arbitrary finite amount of duplication.

## SUMMARY

We described top-down search-based parsing, and described the measure of ambiguity that directly measures efficiency. We demonstrated how much more efficient grammars are that reduce non-determinism, and provide a general mechanism for writing such grammars and converting existing grammars into more efficient forms.

## REFERENCES

[1] SpeechWorks 6.5 Service Creation Guide. 2000. SpeechWorks International, Inc.
[2] J. E. Hopcroft, J. D. Ullman. 1979. Introduction to Automata Theory, Languages and Computation. Addison-Wesley.
[3] Aho, A., R. Sethi, and J. D. Ullman. 1988. Compilers: Principles, Techniques, and Tools. Addison-Wesley.
[4] Jurafsky, D. and J. H. Martin. 2000. Speech and Language Processing. Prentice Hall.
[5] Cormen, T. H., C. E. Leiserson, and R. L. Rivest. 1990. Introduction to Algorithms. MIT Press.
[6] Norvig, P. 1992. Paradigms of Artificial Intelligence Programming. Morgan-Kaufmann.
[7] Johnson, M. 1995. Memoization in Top-down Parsing. Computational Linguistics 21:3, 405-418.


Figure 1. Time vs. Input Length for different codings.


[^0]:    ${ }^{1}$ This includes rejection as a specific instance. Examples include preferences for dates near a particular target such as today, rejecting invalid zip codes, invalid credit card numbers with failed checksums, etc.

[^1]:    ${ }^{5}$ Nuance' s grammar formalism and Sun' s Java Speech Grammar Formalism specification explicitly exclude left recursion from their grammars.
    ${ }^{6}$ It is straightforward to test a grammar; we test for acyclicity of the directed graph with an edge from A to B if there is a rule $\mathrm{A} \Rightarrow \mathrm{B} \ldots$ in the grammar (see [5] for algorithms).
    ${ }^{7}$ In the SpeechWorks grammar formalism, a grammar allowing from one to ten instances of non-terminal $\$ \mathrm{~A}$ would be written as $\$ \mathrm{~A}<1$ $10>$. This abbreviation is then expanded out to an efficient representation.

[^2]:    ${ }^{8}$ We are not considering the cost of maintaining the states' stacks; it is common to reduce this constant cost by precompiling potential state transitions. For instance, this is the principle behind LL parsing (see [3]).
    ${ }^{9}$ Tests were run on a Dell Notebook with a 400 MHz Pentium II, 128 Mb RAM and 256 Kb cache.

